MACBSE: Extracting signals with linear autocorrelations

Zhenwei Shi\textsuperscript{a,*}, Dan Zhang\textsuperscript{b}, Changshui Zhang\textsuperscript{b}

\textsuperscript{a}Image Processing Center, School of Astronautics, Beijing University of Aeronautics and Astronautics, Beijing 100083, P.R. China
\textsuperscript{b}State Key Laboratory of Intelligent Technology and Systems, Department of Automation, Tsinghua University, Beijing 100084, P.R. China

Abstract

This paper proposes blind source extraction methods based on several time-delay autocorrelations of primary sources, called MACBSE. The MACBSE approaches are batch fixed-point learning algorithms for extraction of source signals with linear autocorrelations. The fixed-point algorithms are very simple and do not need choose any learning step sizes. Furthermore, the convergence properties of the algorithms are analyzed. Their efficiencies are demonstrated by extensive computer simulations.

Key words: Blind source separation (BSS); Independent component analysis (ICA); Blind source extraction (BSE); Temporally correlated source

1 Introduction

Blind source separation (BSS) \cite{1,3,5,7,8,11} is an increasingly popular data analysis technique. Generally, there are two principal methods to solve the BSS problem. One approach separates all sources simultaneously. Another approach extracts sources one by one sequentially, called blind source extraction (BSE). The problem of blind (semi-blind) source extraction has received wide attention in various fields such as biomedical signal processing and analysis, data mining, speech and image processing, and so on \cite{6,10}. The BSE may have several advantages over simultaneous blind source separation \cite{6}. For

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* Corresponding author. Tel.: +86-10-627-96-872; Fax: +86-10-627-86-911.
Email address: shizhenwei@mail.tsinghua.edu.cn (Zhenwei Shi).

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example, only “interesting” source signals can be extracted, thus, lots of computing time and resources can be saved; signals can be extracted in a specific order according to some features of source signals.

Many source extraction algorithms can extract a specific signal by using some priori information, such as non-Gaussianity [8], smoothness or linear predictability [2,6], coding complexity [9,14–16], etc. Since many natural signals, like speech signals or biomedical signals have significant temporal structures, it is obvious to use the time-delayed second-order correlations for source extraction. Thus, the work reported in this paper has been motivated by several linear autocorrelations of the desired source signals. An elegant method which is close to the idea, the SOBI algorithm [4], relies only second-order statistics. This algorithm separates all sources simultaneously, which is based on a joint diagonalization of a set of covariance matrices. However, we focus on blind source extraction in the paper and obtain some theoretical results. By capturing the linear autocorrelation characteristics of the signals of interest, we have new insights about BSE.

The structure of the paper is as follows. The objective functions based on the linear autocorrelations of the desired source signals and the fixed-point algorithms for optimizing the objective functions are proposed in Section 2. Furthermore, we prove the convergence properties of the fixed-point algorithms in Section 3. In Section 4, experiments on different datasets are presented. Some conclusions are drawn in Section 5.

2 Proposed Algorithm

2.1 Objective functions

We observe sensor signals $x(t) = (x_1(t), \ldots, x_n(t))^T$ described in the matrix notation:

$$x(t) = As(t),$$

where $A$ is an $n \times n$ unknown mixing matrix, $s(t) = (s_1(t), \ldots, s_n(t))^T$ is a vector of unknown zero-mean and unit-variance primary sources.

A useful preprocessing strategy in BSS is to first whiten the observed variables. This means that we transform the observed vector $x$ linearly so that we obtain a new vector $\tilde{x}$ which is white, i.e., its components are uncorrelated and their variances equal unity. In other words, $x$ is linearly transformed into
a vector
\[ \tilde{x}(t) = \mathbf{V}x(t) = \mathbf{VAs}(t) \] (2)
whose covariance matrix equals the identity matrix: \( E\{\tilde{x}(t)\tilde{x}(t)^T\} = \mathbf{I} \), where \( \mathbf{V} \) is a whitening matrix.

If we want to extract a desired source signal, for this purpose we design a single neural processing unit described as

\[
\begin{align*}
\tilde{y}(t) &= \mathbf{w}^T \tilde{x}(t), \\
\tilde{y}(t - \tau_k) &= \mathbf{w}^T \tilde{x}(t - \tau_k),
\end{align*}
\] (3)
(4)

where \( \mathbf{w} = (w_1, \ldots, w_n)^T \) is the weight vector, and \( \tau_k \) is a certain time delay.

We present the following constrained maximization problem based on linear autocorrelation of the desired source:

\[
\max_{\|\mathbf{w}\|=1} \Psi_1(\mathbf{w}) = E\{\tilde{y}(t)\tilde{y}(t - \tau_k)\} = E\{(\mathbf{w}^T \tilde{x}(t))(\mathbf{w}^T \tilde{x}(t - \tau_k))\}. \tag{5}
\]

The parameter \( \tau_k \) can be chosen as an optimal priori time delay based on some information of the desired source signal. In the cases the optimal time delay can not be achieved, the time delay \( \tau_k \) is often taken to 1.

Sometimes, we could use several time lagged correlations to obtain the desired source signal:

\[
\max_{\|\mathbf{w}\|=1} \Psi_2(\mathbf{w}) = \sum_{k=1}^{M} E\{\tilde{y}(t)\tilde{y}(t - \tau_k)\} = \sum_{k=1}^{M} E\{(\mathbf{w}^T \tilde{x}(t))(\mathbf{w}^T \tilde{x}(t - \tau_k))\}. \tag{6}
\]

Thus, the constrained maximization problem (5) is an especial case of the problem (6) when only one time delay can be used. In Section 3, we give some theoretical properties about the maximization problems (5) and (6).

2.2 Learning algorithms

Maximizing the objective function in (6), we derive a fixed-point BSE algorithm. The gradient of \( \Psi_2(\mathbf{w}) \) with respect to \( \mathbf{w} \) can be obtained as
\[
\frac{\partial \Psi_2(w)}{\partial w} = \sum_{k=1}^{M} E\{ \hat{y}(t - \tau_k) \hat{x}(t) + \hat{y}(t) \hat{x}(t - \tau_k) \}.
\] (7)

According to the Kuhn-Tucker conditions [12], we note that at a stable point of the optimization problem (6), the gradient of \( \Psi_2(w) \) at \( w \) must point in the direction of \( w \), thus we can optimize the objective function in (6) by the fixed-point algorithm [10]. This means that we have

\[
w \leftarrow \sum_{k=1}^{M} E\{ \hat{y}(t - \tau_k) \hat{x}(t) + \hat{y}(t) \hat{x}(t - \tau_k) \},
w \leftarrow w/\|w\|.
\] (8)

Thus, the fixed-point BSE algorithm (MACBSE - Maximizing the AutoCorrelation Blind Source Extraction) is obtained. If only one time delay can be used, i.e., maximizing the objective function in (5), we have

\[
w \leftarrow E\{ \hat{y}(t - \tau_k) \hat{x}(t) + \hat{y}(t) \hat{x}(t - \tau_k) \},
w \leftarrow w/\|w\|.
\] (9)

To estimate several source signals, one can simply use a deflation scheme (Gram-Schmidt orthogonalization scheme) [10]. In experiment section (Section 4) we show the characteristics of the algorithms.

3 Properties of the MACBSE

3.1 Convergence properties of the MACBSE

In this section, we prove some theoretical properties about the MACBSE. First, we give the following stability analysis. \(^1\)

**Theorem 1:** Assume that the input data follows the BSS model (1) with whitened data \( \hat{x} = VA_s \) where \( V \) is the whitening matrix. Furthermore, assume that \( E\{s_s^T\} = I \) and \( \sum_{k=1}^{M} E\{s_s^T \tau_k \} \) is diagonal. Then the local maxima of \( \Psi_2(w) \) under the constraint \( \|w\| = 1 \) include one row of the inverse of the mixing matrix \( VA \) such that the corresponding desired source signal \( s_i \) satisfy

\[^1\] We will drop the sampling index \( t \) for simplicity, i.e., \( y_{\tau_k} = y(t - \tau_k) \) and \( s_{i,\tau_k} = s_i(t - \tau_k) \).
\[
\sum_{k=1}^{M} E\{s_i s_{i,\tau_k}\} > \sum_{k=1}^{M} E\{s_j s_{j,\tau_k}\} \quad (\forall j \neq i).
\]  

(10)

**Proof:** Assume that \(E\{ss^T\} = I\) and the vector \(\tilde{x}\) is white, we have

\[
E\{\tilde{x}\tilde{x}^T\} = VA E\{ss^T\} A^T V^T = (VA)(VA)^T = I,
\]  

(11)

which means the matrix \(VA\) is orthogonal. Make the change of coordinates \(p = (p_1, \ldots, p_n)^T = A^T V^T w\), we have

\[
\Psi_2(p) = \sum_{k=1}^{M} E\{p^T s (p^T s_{\tau_k})\}.
\]  

(12)

Then we can calculate the gradient as

\[
\frac{\partial \Psi_2(p)}{\partial p} = \sum_{k=1}^{M} E\{s(p^T s_{\tau_k}) + s_{\tau_k} (p^T s)\},
\]  

(13)

and the Hessian as

\[
\frac{\partial^2 \Psi_2(p)}{\partial p^2} = \sum_{k=1}^{M} E\{ss_{\tau_k}^T + s_{\tau_k} s^T\}.
\]  

(14)

It is enough to analyze the stability of the point \(p = e_1\), where \(e_1 = (1, 0, 0, \ldots)^T\). Evaluating the gradient and the Hessian at point \(p = e_1\), and using the assumption that \(\sum_{k=1}^{M} E\{ss_{\tau_k}^T\}\) is diagonal, we have

\[
\frac{\partial \Psi_2(e_1)}{\partial p} = 2 e_1 \sum_{k=1}^{M} E\{s_1 s_{1,\tau_k}\}
\]  

(15)

\[
\frac{\partial^2 \Psi_2(e_1)}{\partial p^2} = 2 \sum_{k=1}^{M} \text{diag}(E\{s_1 s_{1,\tau_k}\}, E\{s_2 s_{2,\tau_k}\}, \ldots, E\{s_n s_{n,\tau_k}\})
\]  

(16)

Make a small perturbation \(\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)^T\), we obtain
\[ \Psi_2(e_1 + \varepsilon) = \Psi_2(e_1) + \varepsilon^T \frac{\partial \Psi_2(e_1)}{\partial \varepsilon} + \frac{1}{2} \varepsilon^T \frac{\partial^2 \Psi_2(e_1)}{\partial \varepsilon^2} \varepsilon \]

\[ = \Psi_2(e_1) + 2 \sum_{k=1}^{M} E\{s_is_{1,\tau_k}\} \varepsilon_1 + \sum_{i=1}^{n} \sum_{k=1}^{M} E\{s_is_{i,\tau_k}\} \varepsilon_i^2. \quad (17) \]

Due to the constraint \( \|w\| = 1 \) and the matrix \( VA \) is orthogonal, we have \( \|p\| = \|A^T V^T w\| = 1 \). Thus, we get \( \varepsilon_1 = \sqrt{1 - \varepsilon_2^2 - \varepsilon_3^2 - \ldots - 1} \). Due to the fact that \( \sqrt{1-\alpha} = 1 - \frac{\alpha}{2} + o(\alpha) \), the term of order \( \varepsilon_2^1 \) in (17) is \( o(\|\varepsilon\|^2) \), i.e., of higher order, and can be neglected. Using the first-order approximation for \( \varepsilon_1 \) we obtain

\[ \varepsilon_1 = -\sum_{i>1} \varepsilon_i^2 + o(\|\varepsilon\|^2), \]

which finally gives

\[ \Psi_2(e_1 + \varepsilon) = \Psi_2(e_1) + \sum_{i>1} \left( \sum_{k=1}^{M} (E\{s_is_{i,\tau_k}\} - E\{s_is_{1,\tau_k}\}) \right) \varepsilon_i^2 + o(\|\varepsilon\|^2) \quad (18) \]

which clearly proves \( p = e_1 \) is an extremum, and of the type implied by the condition of the theorem.

If we choose only one time delay, we can similarly prove the following corollary.

**Corollary 1:** Assume that the input data follows the BSS model (1) with whitened data: \( \tilde{x} = VA s \) where \( V \) is the whitening matrix. Furthermore, assume that \( E\{ss^T\} = I \) and \( E\{ss^T_{\tau_k}\} \) is diagonal. Then the local maxima of \( \Psi_1(w) \) under the constraint \( \|w\| = 1 \) include one row of the inverse of the mixing matrix \( VA \) such that the corresponding desired source signal \( s_i \) satisfy

\[ E\{s_is_{i,\tau_k}\} > E\{s_js_{j,\tau_k}\} \quad (\forall \ j \neq i). \quad (19) \]

Next, we prove the convergence properties of the MACBSE. We have the following theorem:

**Theorem 2:** Assume that \( E\{ss^T\} = I \) and \( \sum_{k=1}^{M} E\{ss^T_{\tau_k}\} \) is diagonal, we have the following results.

**A.** Assume that \( \sum_{k=1}^{M} E\{s_is_{i,\tau_k}\} \neq 0 \) and \( \sum_{k=1}^{M} E\{s_js_{j,\tau_k}\} = 0 \) \( (\forall \ j \neq i) \), the MACBSE algorithm (8) converges globally (at the random initial starting points except for those in a zero measure region) after one iteration, this implies that the vector \( w \) converges, up to the sign, to one of the rows of the inverse of the mixing matrix \( VA \).

**B.** Assume that \( |\sum_{k=1}^{M} E\{s_is_{i,\tau_k}\}| > |\sum_{k=1}^{M} E\{s_js_{j,\tau_k}\}| \neq 0 \) \( (\forall \ j \neq i) \), the MACBSE algorithm (8) converges globally (at the random initial starting points except for those in a zero measure region), this implies that the vector
w converges, up to the sign, to one of the rows of the inverse of the mixing matrix VA, and the convergence speed is linear.

**Proof:** (A) To begin with, we make the change of variable \( p = (p_1, \ldots, p_n)^T = A^T V^T w \). From \( E\{ss^T\} = I \) and the property of whitening, the MACBSE algorithm (8) is given by

\[
\hat{p}(r) = \sum_{k=1}^{M} E\{s(p^T(r)s_{\tau_k}) + s_{\tau_k}(p^T(r)s)\}, \quad \text{(20)}
\]

\[
p(r + 1) = \hat{p}(r)/\|\hat{p}(r)\|, \quad \text{(21)}
\]

where \( r \) means iteration (\( r = 0 \) means initial value). Using the assumption that \( \sum_{k=1}^{M} E\{ss_{\tau_k}^T\} \) is diagonal and doing some algebraic manipulations, we obtain

\[
\hat{p}_i(0) = \sum_{k=1}^{M} E\{s_i(p_1(0)s_{1,\tau_k} + p_2(0)s_{2,\tau_k} + \ldots + p_n(0)s_{n,\tau_k})
+ s_{1,\tau_k}(p_1(0)s_1 + p_2(0)s_2 + \ldots + p_n(0)s_n)\}
= 2p_i(0) \sum_{k=1}^{M} E\{s_is_i_{\tau_k}\}, \quad \text{(22)}
\]

\[
\hat{p}_j(0) = \sum_{k=1}^{M} E\{s_j(p_1(0)s_{1,\tau_k} + p_2(0)s_{2,\tau_k} + \ldots + p_n(0)s_{n,\tau_k})
+ s_{1,\tau_k}(p_1(0)s_1 + p_2(0)s_2 + \ldots + p_n(0)s_n)\}
= 2p_j(0) \sum_{k=1}^{M} E\{s_js_{j,\tau_k}\}, \quad \text{(23)}
\]

where \( \forall j \neq i \). Assume that \( \sum_{k=1}^{M} E\{s_is_{i,\tau_k}\} \neq 0 \) and \( \sum_{k=1}^{M} E\{s_js_{j,\tau_k}\} = 0 \) (\( \forall j \neq i \)), after the normalization, we can see the vector \( p(1) \) become such a vector \( e \) that \( e_i = \pm 1 \) and \( e_j = 0, \forall j \neq i \). That is, the algorithm converges after only one iteration at the random initial starting points except for those points satisfied \( p_i(0) = 0 \) (a zero measure region), i.e., the vector \( w = ((VA)^T)^{-1} p \) converges, up to the sign, to one of the rows of the inverse of the mixing matrix VA.

(B) On the other hand, assume that \(|\sum_{k=1}^{M} E\{s_is_{i,\tau_k}\}| > |\sum_{k=1}^{M} E\{s_js_{j,\tau_k}\}| \neq 0 \) and \( p_l(0) \neq 0 \) (\( \forall l \)), from (22) and (23), we obtain:

\[
\frac{|\hat{p}_i(r)|}{|\hat{p}_j(r)|} = \frac{|p_i(r)||\sum_{k=1}^{M} E\{s_is_{i,\tau_k}\}|}{|p_j(r)||\sum_{k=1}^{M} E\{s_js_{j,\tau_k}\}|}, \quad \text{(24)}
\]
We see that all the other component $\hat{p}_j(r)$ ($\forall j \neq i$), quickly become small compared to $\hat{p}_i(r)$ as the iteration $r$ increases. Taking the normalization $\|\hat{p}(r)\| = 1$ into account, this means that $|p_i(r)| \rightarrow 1$ and $p_j(r) \rightarrow 0$ ($\forall j \neq i$). This implies that $w$ converges as the theorem stated. The equation (24) shows that the convergence is linear.

We prove that the convergence of the MACBSE algorithm (8) is global except for a zero measure region, however, we can safely assume that in practice, the initial starting points do not lie in the zero measure region. Again, if we choose only one time delay, we have the following corollary.

**Corollary 2**: Assume that $E\{ss^T\} = I$ and $E\{ss_{\tau_k}^T\}$ is diagonal, we have the following results.

A. Assume that $E\{s_is_{\tau_k}\} \neq 0$ and $E\{s_j\tau_{\tau_k}\} = 0$ ($\forall j \neq i$), the MACBSE algorithm (9) converges globally (at the random initial starting points except for those in a zero measure region) after one iteration, this implies that the vector $w$ converges, up to the sign, to one of the rows of the inverse of the mixing matrix $VA$.

B. Assume that $|E\{s_is_{\tau_k}\}| > |E\{s_j\tau_{\tau_k}\}| \neq 0$ ($\forall j \neq i$), the MACBSE algorithm (9) converges globally (at the random initial starting points except for those in a zero measure region), this implies that the vector $w$ converges, up to the sign, to one of the rows of the inverse of the mixing matrix $VA$, and the convergence speed is linear.

### 3.2 Further analysis of the MACBSE

In fact, the MACBSE can be very simple iteration, to see this, the algorithm (8) can be written as

$$w \leftarrow \sum_{k=1}^{M} E\{\tilde{x}(t)\tilde{x}(t-\tau_k)^T + \tilde{x}(t-\tau_k)\tilde{x}(t)^T\}w,$$

$$w \leftarrow w/\|w\|.$$  \hspace{1cm} (25)

This means we can compute the matrix $R = \sum_{k=1}^{M} E\{\tilde{x}(t)\tilde{x}(t-\tau_k)^T + \tilde{x}(t-\tau_k)\tilde{x}(t)^T\}$ first, and circulate

$$w \leftarrow Rw,$$

$$w \leftarrow w/\|w\|.$$  \hspace{1cm} (26)

Although the proof shows the convergence speed of the algorithm is linear under some conditions, we can see that this algorithm is a very simple and
compute efficiently due to the low computation load.

From the theorems, we can obtain the following further properties of the MACBSE:

(a) If the sensor signals are linearly mixed by one temporal correlated signal and some i.i.d. source signals, the one-unit MACBSE algorithms can extract the temporal correlated signal using only one iteration.

(b) The extracted temporal signals using the MACBSE algorithm (9) (or algorithm (8)) with the deflation scheme are in a decreasing order according to absolute values (or sum of absolute values) of time-delay autocorrelations of the source signals.

(c) From mixtures of temporal correlated signals and i.i.d. signals, only temporal correlated signals can be extracted from the mixtures using the MACBSE algorithms with the deflation scheme.

4 Experimental Results

4.1 Experimental results about property (a)

To begin with, we validate the interesting character of “one-iteration” convergence of the MACBSE algorithms. Three 128 \times 128 images are used, with a natural image used for Source 2. This image has temporal correlations when scanned in one dimension. The other two images are interferences artificially generated from two Gaussian i.i.d. noises. The images are mixed with a randomly 3 \times 3 mixing matrix. Figure 1 shows, from top to bottom, the original images Source 1, Source 2 and Source 3, the mixed images Mixture 1, Mixture 2 and Mixture 3, and the extracted image after only one iteration using the learning algorithm (8) (\tau_k = 1, \ldots, 10, M = 10). The normalized kurtosis of Source 1, Source 2 and Source 3 are -0.032, -0.58 and 0.023, respectively. The experiment is independently repeated 100 times, the averaged normalized kurtosis of the extracted image is -0.58. As can be seen from Figure 1, this natural image has been successfully extracted. We also run the algorithm (9) (\tau_k = 1), the averaged normalized kurtosis of the extracted image is -0.56.

Remark: It is worth noting that due to the effect of finite samples, linear autocorrelations of the sources are generally non-zero, thus the better results can be achieved after two iterations (the averaged normalized kurtosis is just -0.58).
Fig. 1. Simulation results for mixture of three 128 × 128 image signals. From top to bottom, the original images Source 1, Source 2 and Source 3, the mixed images Mixture 1, Mixture 2 and Mixture 3, and the extracted image after only one iteration using the learning algorithm (8).

4.2 Experimental results about property (b)

In the second experiment, we generate four colored Gaussian signals (50000 samples).

\[
\begin{align*}
    s_1(t) &= 0.1 \ast s_1(t - 1) + v_1(t),
    \quad (27) \\
    s_2(t) &= 0.5 \ast s_2(t - 1) + v_2(t),
    \quad (28) \\
    s_3(t) &= -0.6 \ast s_3(t - 1) + v_3(t),
    \quad (29) \\
    s_4(t) &= 0.9 \ast s_4(t - 1) + v_4(t),
    \quad (30)
\end{align*}
\]

where \( v_i \) is white Gaussian signals \( \sim N(0, 1) \). The time-delay autocorrelations \( (\tau_k = 1) \) of the source signals are 0.10 \( (s_1) \), 0.67 \( (s_2) \), -0.96 \( (s_3) \) and 4.69 \( (s_4) \), respectively. The source signals are randomly mixed by a 4 × 4 mixing matrix. In order to measure the accuracy of extraction, we adopt the performance index:

\[
PI = -10 \log E\{(s(t) - \hat{s}(t))^2\},
\]

(31)

where \( s \) is the original signal, and \( \hat{s} \) is the extracted corresponding signal (both are normalized to be zero-mean and unit-variance). The higher PI is,
the better the performance is.

We run the algorithm (9) (\(\tau_k = 1\)) with the deflation scheme. Figure 2 shows, from top to bottom, the original source signals \(s = [s_1, s_2, s_3, s_4]^{T}\), the mixed signals \(x = [x_1, x_2, x_3, x_4]^{T}\), and the extracted source signals \(y = [y_1, y_2, y_3, y_4]^{T}\). The mean values of the performance indexes of 100 independent trials by the algorithm are 51.19 (\(s_1\)), 47.07 (\(s_2\)), 40.99 (\(s_3\)) and 40.77 (\(s_4\)), respectively. Interestingly, the extracted temporal signals are in a decreasing order according to absolute values of time-delay autocorrelations of the source signals. Both the performance indexes and a visual comparison between the original source signals and the extracted signals confirm the validity of the property (b).

4.3 Experimental results about property (c)

In the third experiment, one voice signal (wqyn_voice.wav) and one music signal (wqyn_organ.wav) (taken from [18]) (available at http://www.au.tsinghua.edu.cn/szll/bodao/ zhangchangshui/ bigeye/ member/ zyghtm/ Experiments.htm), and two i.i.d. Gaussian signals are mixed by a randomly \(4 \times 4\) mixing matrix. Our aim here is to show a property of the MACBSE algorithms, which are able to extract only temporal correlated signals. We apply the algorithm (8) (\(\tau_k = 1, \ldots, 10, M = 10\)) on the mixed signals. From mixtures of temporal
Fig. 3. Simulation results for mixture of one voice signal, one music signal, and two i.i.d. Gaussian signals. The original source signals $s_1, s_2, s_3, s_4$, the mixed signals $x_1, x_2, x_3, x_4$, and the extracted source signals $y_1, y_2$ from the first two extraction processing units using the learning algorithm (8).

correlated signals and i.i.d. signals, we can see that only temporal correlated signals can be extracted from the mixtures. Figure 3 shows, from top to bottom, the original source signals $s = [s_1, s_2, s_3, s_4]^T$, the mixed signals $x = [x_1, x_2, x_3, x_4]^T$, and the extracted source signals $y = [y_1, y_2]^T$ from the first two extraction processing units. The mean values of the performance indexes of 100 independent trials by the algorithm are 58.04 (wqyn_voice.wav) and 34.19 (wqyn_organ.wav). For comparison, we run the algorithm (9) ($\tau_k = 1$) on the test datasets. The mean values of the the performance indexes of 100 independent trials are 36.31 (wqyn_voice.wav) and 29.53 (wqyn_organ.wav). Both the performance indexes and a visual comparison between the original source signals and the extracted signals confirm the validity of the property (c).

4.4 Compared with the related BSE algorithm

A method which is close to ours is an important and simple BSE algorithm developed by Barros and Cichocki [2] (for simplicity, we call it BCBSE) as follows:
\[ \mathbf{w} \leftarrow E\{\tilde{y}(t - \tau_k)\tilde{x}(t)\}, \]
\[ \mathbf{w} \leftarrow \mathbf{w}/\|\mathbf{w}\|. \]  

(32)

The first order linear predictor of primary sources and only one lag are used by the work. For comparison, we run the algorithm using \(\tau_k = 1\) on the second test datasets (mixture of four colored Gaussian signals). The mean values of the the performance indexes of 100 independent trials are 47.04 (s_1), 44.85 (s_2), 36.21 (s_3) and 23.13 (s_4), respectively. Clearly, the MACBSE algorithm (9) performs better.

It is worth noting that we can also similarly prove the BCBSE (32) has the same convergence as the MACBSE algorithm (9), in fact, neglecting the difference between \(E\{\tilde{x}_\tau \tilde{x}_\tau^T\}\) and \(E\{\tilde{x}\tilde{x}_\tau^T\}\), we can see that the MACBSE algorithm (9) is just the BCBSE (32) (note that the original BCBSE is derived from the linear predictor). However, from the experiment results, it can be seen that the MACBSE algorithm (9) is a robust version.

4.5 Experiments on the extraction of fetal electrocardiogram

One important real-world application of BSE is the extraction of fetal electrocardiogram (FECG), the aim is to obtain the clear FECG as the first extracted signal [2,16]. The FECG contains valuable information about the health and condition of the fetus. However, it is always corrupted by various kinds of noise, such as the maternal electrocardiogram (MECG) with extreme high amplitude, respiration and stomach activity, thermal noise, noise from electrode-skin contact, etc.

The BCBSE (32) provides a method for the extraction of FECG, which has been proposed in the paper [2]. It is in essence based on the linear autocorrelation of FECG. However, from the following experiments, we can see that the FECGs extracted by the algorithms based on linear autocorrelations (also concluding the MACBSE algorithms) often contain noise contributions (such as the mother’s breathing artifact), and are very sensitive to the estimation error of the time delay \(\tau_k\). From the proposed theorems, we can gain some insights why the BSE methods based on the linear autocorrelations do not work well. Thus, we should take care to use the BSE methods for the extraction of FECGs.

We perform experiments on real-world ECG data – the well-known ECG measured from a pregnant women and distributed by De Moor [13]. Figure 4 shows that 10 seconds of recordings resampled at 250 Hz [2]. The aim is to only extract the desired FECG signal. By carefully examining the autocorrelation of the sensor signal in channel 1 (where the fetal influence is clearly stronger
Fig. 4. ECG signals measured from a pregnant women.

Fig. 5. Comparison of the extracted FECGs at the optimal time delay $\tau = 112$. (F1,F2,F3) are extracted by the BCBSE, the MACBSE ($M = 1$), and the MACBSE ($M = 10$), respectively.

than in the other signals) and using the fact that the fetal heart should strike every 0.5 second or so, we find that it has a peak at $\tau_k = 112$ (i.e., 0.448 second) (for details, please see the paper [2]). We initialize the weight by $w = (1,0,0,\ldots)^T$ in the BSE algorithms and use $\tau_k = 112$ as the optimal time delay for extracting the desired FECG. The FECGs extracted by the BCBSE, the MACBSE ($M = 1$), and the MACBSE ($M = 10$), are shown in Figure 5 (F1,F2,F3, respectively). Compared with the BCBSE, the MACBSE methods obtain clearer FECGs. However, the all extracted FECGs contain some contributions of the mother’s breathing artifacts.
If the estimated optimal time delay $\tau_k$ has some errors, which cannot be avoided in practice, the situation is even more serious. To see it, we apply the BSE algorithms on the same ECG data, but adopt different time delay $\tau_k$. The results are plotted in Figure 6 ($\tau_k = 113$) and Figure 7 ($\tau_k = 114$). From the results, we can see the linear autocorrelations methods are very sensitive to the estimation error of the time delay $\tau_k$. The BCBSE fails to estimate the FECG at $\tau_k = 113$ and the three algorithms all fail to estimate the FECG at $\tau_k = 114$. In fact, the extracted signals at $\tau = 114$ by the three algorithms are even similar to the mother’s breathing artifacts.

From the proposed theorems, we obtain the theoretical reason why the
BSE methods based on linear autocorrelations do not perform well. As we adopt the optimal time delay ($\tau_k = 112$) of the FECG, the linear autocorrelations of the other sources (such as mother’s breathing artifact) at $\tau_k = 112$ could be also high (close to the value of the desired FECG). Thus, the performance of linear autocorrelations methods degrades. If we adopt the other time delays, the linear autocorrelations of the noise contributions (such as mother’s breathing artifact) could be even higher than that of the desired FECG. Thus, the extracted signals by the BSE algorithms could be more similar to the mother’s breathing artifacts. It makes the methods fail to extract the FECG in the cases. The results imply that the linear autocorrelations could not characterize the temporal structures of the desired FECG well, one could need to use other more suitable temporal structures for the extraction of FECCGs. Some methods have been proposed, such as the BSE algorithm in the paper [16].

5 Conclusions

We have proposed the MACBSE algorithms for blind source extraction based on autocorrelations of the source signals and have analyzed the convergence properties of the algorithms. Although the theorems show that the convergence speeds of the algorithms are linear, the algorithms in fact compute efficiently due to the low computation load. It is worth noting that our algorithms obtained here are very simple and do not need choose any learning step sizes. However, it is also worth noting that we should choose the linear autocorrelations methods for extracting the desired signals in suitable applications, based on the stability conditions presented in our paper. In some cases, such as the extraction of FECCGs, we do not suggest to use the BSE methods based on the linear autocorrelations. Furthermore, the selection of $\tau_k$ and $M$ in the MACBSE algorithms is a nontrivial task in some applications. The theoretical conditions about the selection of $\tau_k$ and $M$ should be considered in future research.

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References


